Exploring Ranking Theory as an Alternative Model for Human Uncertainty Representation

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What is "Ranking Theory"?

- A normative belief revision model that represents degrees of belief and disbelief, as an alternative to probability theory[1,2].
- To investigate ranking theory as an alternative to probabilistic approaches, four experiments (E1:E4) examined the relationship between ranking functions and subjective probabilities.

Metrics of beliefs

A grading of **disbelief** (or surprise) expressed by ***negative*** *ranking function,* κ : *(scale from 0 to* ∞*)*[1,2]

$\kappa(A) = 0 : A$ is not disbelieved (not surprising) (1)

$\kappa(A) > 0 : A$ is disbelieved (surprising) (2)

$\kappa(A) = \infty : A$ is considered impossible (3)

$\kappa(A) = 0$ or $\kappa(\bar{A}) = 0$: [the law of negation] (4)

A grading of **belief** expressed by ***positive*** *ranking function,* β : *(scale from 0 to* ∞*)*[1,2]

$\beta(A) = \kappa(\bar{A})$: Belief in A equals disbelief in not-A (5)

Integrating *positive* and *negative* ranks into a ***two-sided*** ranking function τ expresses **belief and disbelief** at once: *(scale from -* ∞ *to* ∞*)*[1,2]

$\tau(A) = \beta(A) - \kappa(A) = \kappa(\bar{A}) - \kappa(A)$ (6)

Probability-Rank translation

$\kappa(A) = \log_b P(A) - \log_b \max_{S \in \Omega} P(S), \quad b \in (0,1)$ (7)

$P(A) + P(\bar{A}) = 1$ (8)

$\tau(A) = \kappa(\bar{A}) - \kappa(A) = \log_b(1 - P(A)) - \log_b P(A)$ (9)

$\tau(A) = \log_b(\frac{1-P(A)}{P(A)})$: [probability to two-sided rank] (10)

$P(A) = \frac{1}{b^{\tau(A)}+1}$: [two-sided rank to probability] (11)

E1:E3 Methods - unknown objective probability

Grade your degrees of **disbelief:**

(reminder: ***0*** *for* ***not surprising****, value* ***greater than 0*** *for* ***surprising****, with infinity for impossible)*

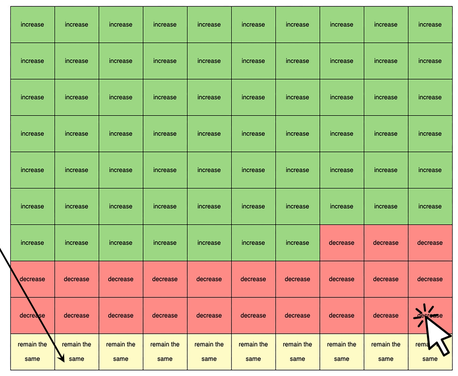
Real Estate Investing:

Suppose you buy a property in Toronto, Ontario today. How **surprised** would you be if the property's value _____ 5 years from today?

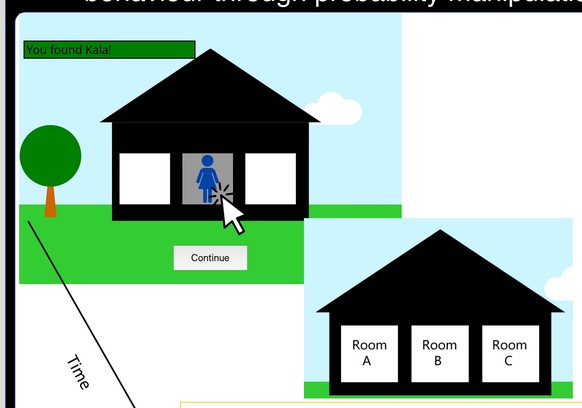
increased ☐

decreased ☐

remained the same ☐



E4 Methods - learning task: modelling opponent behaviour through probability manipulation



Grade your degrees of *disbelief*:

(reminder: ***0*** *for* ***not surprising****, value* ***greater than 0*** *for* ***surprising****, with infinity for impossible)*

How **surprised** would you be if Kala was hiding in _____?

Room A ☐

Room B ☐

Room C ☐

Participants' use of negative ranking functions to express disbelief was consistent across different propositions

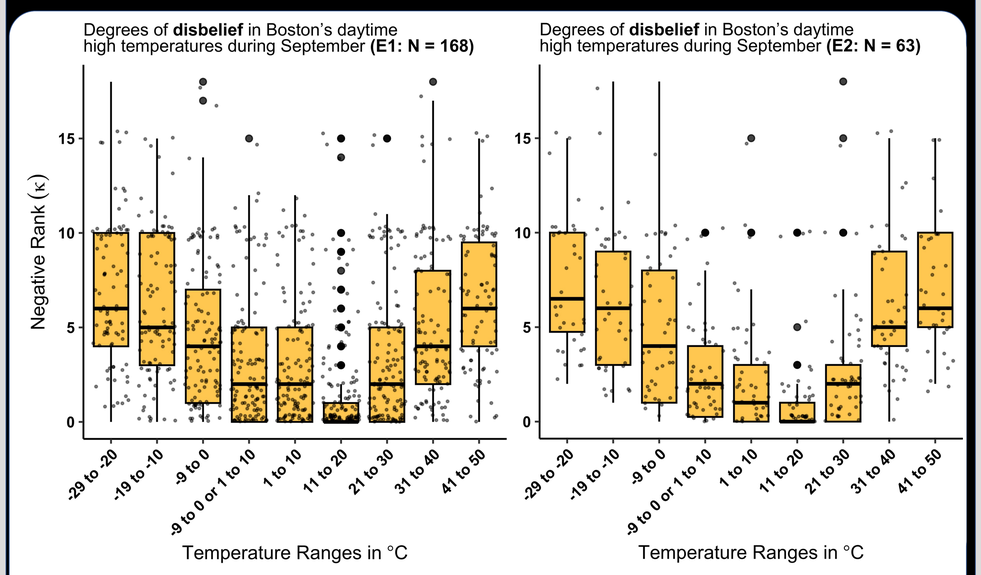


Fig. 1. Participants assigned numerical values to their degrees of disbelief toward a set of propositions presented in a short questionnaire with unknown objective probabilities. This figure shows participants' disbelief towards temperature ranges for Boston's daytime high temperatures during September. Experiment 2 replicated the findings from Experiment 1's consistent negative ranking values towards propositions, represented from a scale from 0 to ∞. Participants found the temperature range of 11 to 20°C to be least surprising.

Majority of participants obey the law of negation and can use positive ranking functions to grade degrees of belief

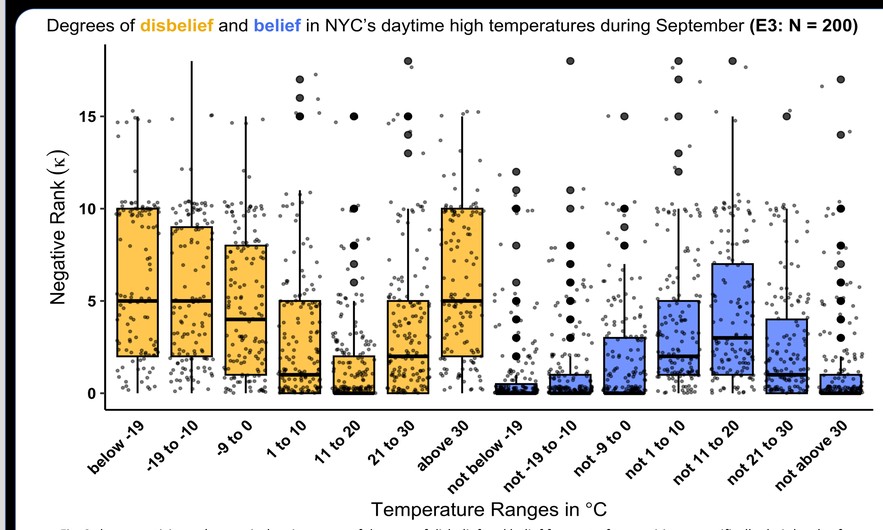


Fig. 3 shows participants' numerical assignments of degrees of disbelief and belief for a set of propositions, specifically their levels of disbelief and belief toward temperature ranges for New York City's daytime high temperatures during September. The temperature range of 11 to 20°C, like E1 and E2, was found to be the least surprising. Positive ranks were measured as disbelief in the negation of a proposition (Eq. 5). Moreover, most participants adhered to the law of negation, which means they did not disbelieve in both the proposition and its negation (for example, disbelief in "11 to 20°C" and disbelief in "not 11 to 20°C" would violate the law of negation).

Exploring the logarithmic relationship: degrees of disbelief and subjective probability in the hide-and-seek learning task

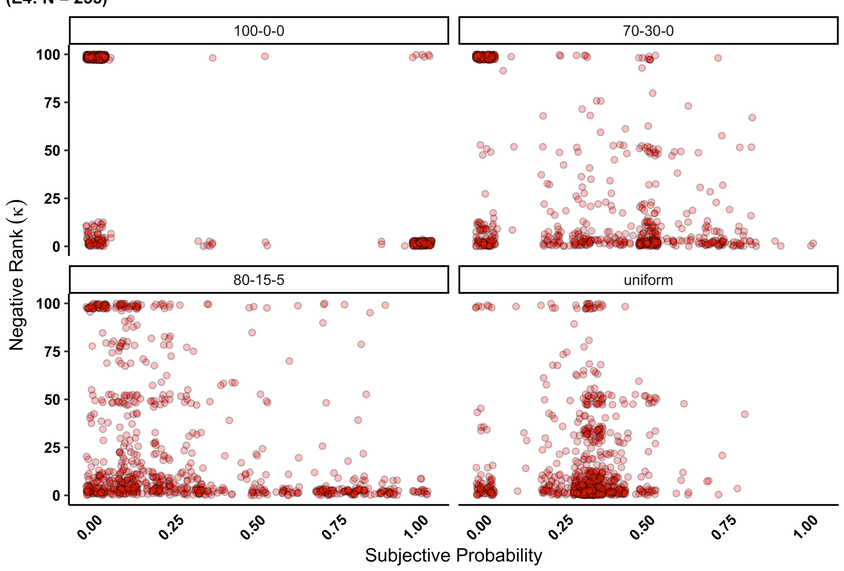


Fig. 5. Participants assigned numerical values to their degrees of disbelief towards each room after playing a hide-and-seek game with opponents having objective probability distributions of 100-0-0, 70-30-0, 80-15-5, and uniform. The inclusion of the p = 0 room condition aimed to elicit maximum surprise. The results reveal a consistent pattern indicating a logarithmic relationship, particularly prominent in the 80-15-5 distribution condition. It is worth noting that while some participants reported a p = 0, a distinct group of participants exhibited differing levels of surprise towards the p = 0 room condition, ranging from minimal surprise (or not surprising) to extreme surprise (or was considered impossible).

References

1. Spohn, W. (2013). A ranking-theoretic approach to conditionals. *Cognitive Science*, *37*(6), 1074-1106.
2. Skovgaard-Olsen, N. (2016). Ranking theory and conditional reasoning. *Cognitive Science*, *40*(4), 848-880.
3. Tiede, K. E., Henninger, F., & Kieslich, P. J. (2022). Revisiting the Open Sampling format: Improving risky choices through a novel graphical representation. *Psychonomic Bulletin & Review*, 1-12.

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Disbelief thresholds represented by subjective probability vary depending on context

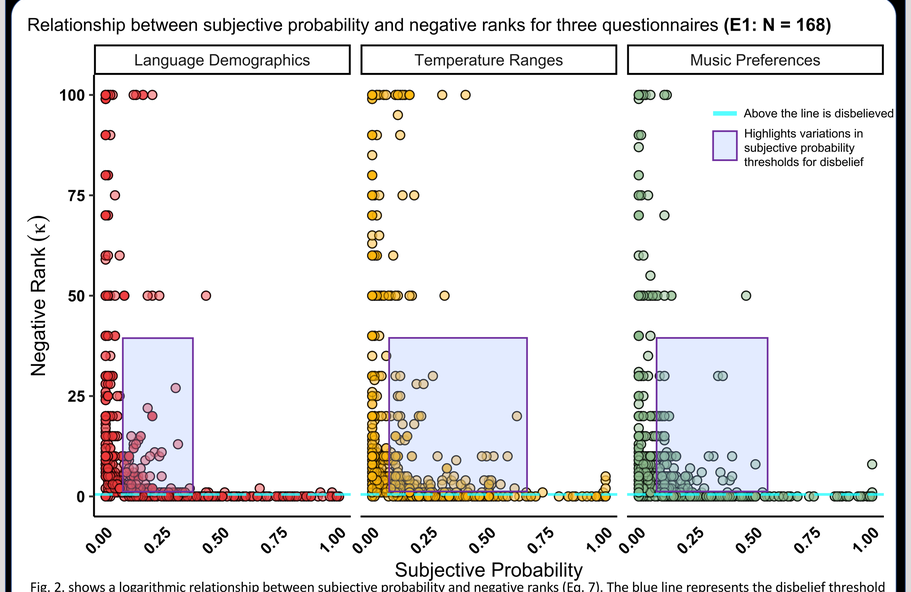

Fig. 2. shows a logarithmic relationship between subjective probability and negative ranks (Eq. 7). The blue line represents the disbelief threshold (Eq. 2), and values below it indicate that the proposition is not disbelieved and may represent some degree of belief (Eq. 1). For propositions that were not disbelieved, participants provided the entire range of subjective probability ranging from 0 to 1. The purple box highlights varying subjective probability thresholds for disbelief, indicating context dependence. Overall, this suggests that subjective probability alone may not be sufficient to explain degrees of belief and disbelief.

Log-odds relationship between probability and two-sided ranks

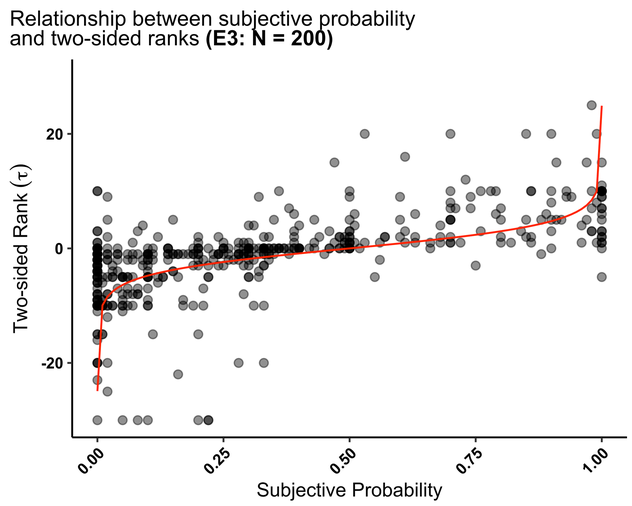

Fig. 4. illustrates a log-odds relationship between subjective probability and two-sided ranks (red line; Eq. 10). The two-sided ranks for participants' real estate investment questionnaire was computed using Eq.6.

Degrees of disbelief represent objective probability distribution with greater accuracy compared to subjective probability

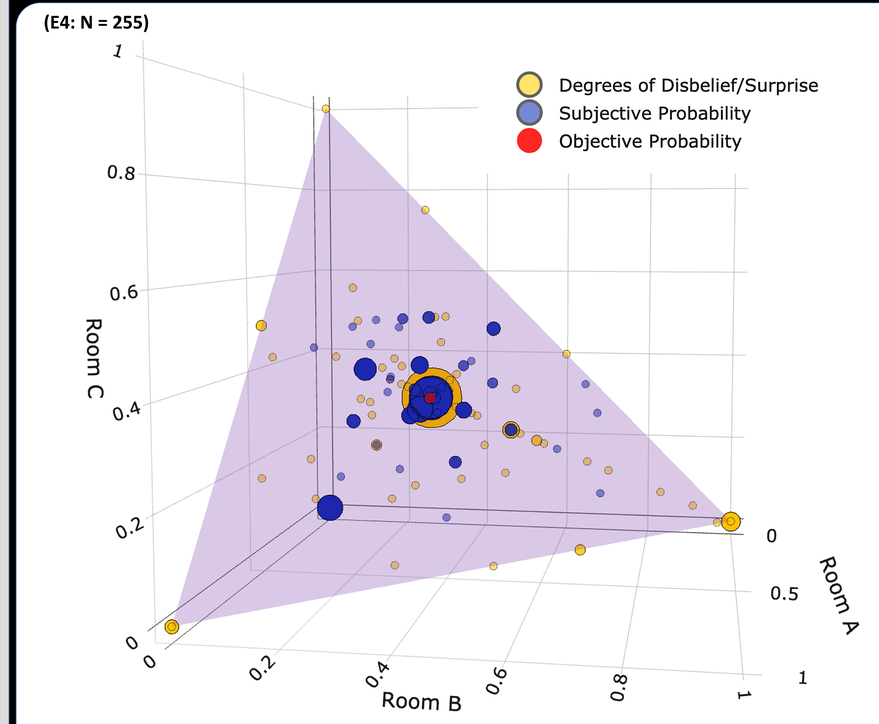

Fig. 6. A set of probability vectors forms the probability simplex, Δ^{n-1}, a generalized triangle. The translated ranks were closer to the objective probabilities (**115 correct responses**) compared to subjective probabilities (**61 correct responses**). Subjective probability responses provided by participants were less accurate compared to degrees of disbelief/surprise when mapped onto the simplex, enabling a direct comparison with objective probability. Note that the size of the points represents the frequency of occurrence in the respective x, y, z coordinates.